Branch-and-Price for a European variant of the Railroad Blocking Problem

Robert Voll 1

Institute of Transport Logistics
TU Dortmund
Germany

Uwe Clausen 2

Fraunhofer Institute for Material Flow and Logistics
& Institute of Transport Logistics
TU Dortmund
Germany

Abstract
In wagonload traffic, a production form in railway freight traffic, small groups of wagons have to be transported. In order to decrease transportation costs, wagons from different relations are consolidated. In railyards trains can be separated and rearranged to new trains. The costs arising from this process, which is called re-classification, must be balanced with the transportation costs. The minimization of total costs can be formulated as a network optimization model. We introduce a branch-and-price approach for the considered problem. Moreover, we present specialized cuts, which can be incorporated into the branching scheme. Solutions obtained from our implementation can keep up with results computed by CPLEX.

Keywords: column generation, branch-and-price, railway freight traffic, optimization, railroad blocking problem
1 Introduction

Railway is one of the most important modes of transport world-wide. Despite all liberalization efforts and a stronger awareness of ecological needs, railroad freight traffic was not able to increase its share in European modal split significantly in the last decades. Especially wagonload traffic is confronted with strong economic issues. In wagonload traffic a customer orders the transport of single railcars or a small group of railcars. Due to the small number of railcars, direct transport is rarely economic. Therefore, it is reasonable to consolidate cars from different relations, i.e., origin-destination pairs, to decrease transportation costs. Trains can be separated and rearranged to new trains in reclassification yards [5]. This process also results in high costs. The sequence of yards for each railcar is determined by so called blocking plans. Transportation and reclassification costs depend on the blocking plan.

The generation of efficient blocking plans can be modelled as a large-scale network optimization problem and is referred to as the Railroad Blocking Problem (RBP). The resulting models have a multicommodity flow structure with elements of network design problems. We extend a column generation approach published earlier [12] to a branch-and-price algorithm (BaP).

The remainder of this paper is structured as follows: Section 2 gives a very short survey on optimization of blocking plans. In section 3 our model is presented, followed by a description of our BaP approach in 4. Afterwards, we introduce cuts for the presented model in 5. Section 6 provides numerical results. The paper is concluded by a summary.

2 Survey

Several contributions in the field of blocking plans deal with North American railway systems ([1,4,7,11]). In contrast to Europe, the networks are characterized by a strict separation of passenger and freight traffic. Due to mainly infrastructural reasons, freight trains are limited in total length and weight in Europe. Consequently, corresponding constraints are incorporated into models designed for application in Europe ([6,9]). A Model for Iranian([13]) railways has recently been published. The differences between North American and Chinese planning strategies are explained in [10].

Most of the solution approaches are customized to the particular models and can hardly be transferred to other models.

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1 Email: voll@itl.tu-dortmund.de
2 Email: uwe.clausen@iml.fraunhofer.de
3 Model

The RBP can be modeled as a multicommodity capacitated network design problem (MCNDP). The underlying problem of determining a sequence of yards for each railcar is a routing problem. Nevertheless, it has strong network design characteristics. The considered graph $G$ is complete and consists of the reclassification yards $N$ and railway connections $A$ between them. While finding an optimal routing for all railcars, we implicitly want to find the so called blocking network, which is a subset of the physical network, see graphic 1. The arcs of this blocking network are established by trains in the physical network.

![Graphic 1: blocking network](image)

Operating a train induces high fixed costs, which dominate the transport costs in European wagonload traffic. So, the cost function must focus rather on train costs than on costs per wagon. Hence, we introduce two kinds of decision variables: path variables $\lambda^k_p$ and design variables $n_{ij}$. The path variables select a particular path $p$ for relation $k$, i.e. wagons with a common origin $o(k)$ and destination $d(k)$, from the set of all feasible paths $P(k)$. A path is feasible for a relation, if it connects origin and destination of the relation and does not traverse more than $S_k$ arcs. The design variables give the number of trains established on each arc.

$$\lambda^k_p = \begin{cases} 1, & \text{if relation } k \text{ uses path } p \in P(k) \\ 0, & \text{otherwise} \end{cases} \quad (1)$$

$$n_{ij} = \text{number of trains established on arc (i,j)} \quad (2)$$

Moreover, some parameters are necessary. Due to aspects of robustness, we do not look for a solution for a single day, but for a blocking plan, which works well on a couple of days $D$. Accordingly, particular demand parameters for each day of the planning horizon are given. Further information about this
setup can be found in [6]. A path \( p \in \mathcal{P}(k) \) can be characterized by the arcs \((i,j)\) it traverses. So, \( x_{ij}^k \) is one, iff \((i,j)\) is an arc on path \( p \in \mathcal{P}(k) \). The transportation costs for a train on arc \((i,j)\) and the classification costs per wagon in yard \( i \) are given by \( c_{ij} \) and \( U_i \), respectively. The parameters \( v^d_k, \lambda^d_k \), and \( w^d_k \) give the number of wagons on relation \( k \) on day \( d \) and their aggregated lengths and weights, respectively. \( W_{ij} \) is the maximal train weight allowed on \((i,j)\) and, accordingly, \( L_{ij} \) the maximal length. Parameters \( R_i \) restrict the number of trains which can be built in yard \( i \) per day. The following optimization model \((OPT_{path})\) is defined:

\[
\min_{x,n} \sum_d \sum_{i,j} \left[ c_{ij} n_{ij} + \sum_k U_i \lambda^d_k x_{ij}^k \right] \tag{3}
\]

s.t.

\[
\sum_k l^d_k \lambda^d_k x_{ij}^k \leq L_{ij} n_{ij} \quad \forall d \in D \quad \forall (i,j) \in A \tag{4}
\]

\[
\sum_k w^d_k \lambda^d_k x_{ij}^k \leq W_{ij} n_{ij} \quad \forall d \in D \quad \forall (i,j) \in A \tag{5}
\]

\[
\sum_j n_{ij} \leq R_i \quad \forall i \in N \tag{6}
\]

\[
\lambda^d_k \in \{0,1\} \quad \forall k \in K \quad \forall p \in \mathcal{P}(k) \tag{7}
\]

\[
n_{ij} \in \mathbb{N} \quad \forall (i,j) \in A \tag{8}
\]

The objective function sums up train and reclassification costs. Constraints (4) and (5) reflect the aforementioned restrictions on trains length and weight, respectively. The reclassification capacity of each node is measured by the number of outgoing trains in (6). All path variables must be binary, because relations shall not be splitted, and the number of trains on an arc should be integral. The model \((OPT_{path})\) is equivalent to a compact arc-based formulation \((OPT_{arc})\) with arc-flow variables \( x_{ij}^k \). The following transformation shows their equivalence.

\[
x_{ij}^k = \sum_{p \in \mathcal{P}(k)} \lambda^d_k x_{ij}^k \tag{9}
\]

The decision problem corresponding to \((OPT_{path})\) can be proven to be \( \mathcal{NP} \)-complete, because it includes the integer multicommodity flow problem (IMCFP) as a special case. Following Garey and Johnson ([8]), IMCFP is known to be \( \mathcal{NP} \)-complete even for two commodities.
4 Branch-And-Price

(*OPT* _path_) can hardly be solved by standard solvers. Small instances can be solved to optimality, but medium size instances (up to ~ 250 relations) result in excessively high run times. For real-world instances, it was not even possible for CPLEX 12.4 to solve the LP in the root node. So, we developed a column generation (CG) algorithm in [12], which was reasonable, because our problem has a block diagonal structure, which is vital to CG (e.g. [3]).

A set of paths for each relation could be generated in polynomial time using the Bellman-Ford shortest path algorithm. Our implementation outperforms the CPLEX LP solver, although it was not able to solve real-world instances, either. However, we are not (only) interested in LP solution, but want to solve the integer problem. So, it is necessary to embed CG into a BaP framework.

The success of BaP strongly depends on the branching decisions. Thus, we want to discuss, which branching is reasonable for (*OPT* _path_). The branching must incorporate two aspects. On the one hand, the branching should have an impact on the objective function value of the LP. Otherwise the size of the branching tree can hardly be restricted. On the other hand, decisions should lead the search into branches with promising solutions. Therefore, it is important to eliminate fractional solutions. Considering the structure of (*OPT* _path_), a branching on the train number variables _n_ _ij_ seems reasonable, because of their strong impact in the objective function. Unfortunately, our tests showed that it does not help to eliminate fractional solutions. Hence, no good primal solutions were found. Branching on the path variables has undesired effects, which lead to an unsymmetric partition of the decision tree, as stated in [2]. Using transformation (9), the equivalence of compact and extensive formulation, we can branch on the arc-flow variables _x_ _kij_ of the compact formulation.

Fixing a variable _x_ _kij_ to zero is easy. We can simply delete the arc from the graph, when generating new paths. Fixing _x_ _kij_ = 1 is complicated, because it forces a relation to use the particular arc (_i, j_). The resulting constrained shortest path problem cannot be solved in polynomial time anymore. We overcome this problem by solving the pricing problem by dynamical programming instead of Bellman-Ford algorithm. This is not difficult, because paths, which must include certain arcs, can be composed from regular shortest paths.

One more decision is important for the implementation of Branch-and-Price: Which sequence of _x_ _kij_ is used for branching? We follow an idea of Barnhart et al. ([2]). The largest relation _k_ _max_ with fractional path variables _λ_ _k_ _max_ is chosen. The size of a relation can be measured by the number of trains induced by it. It can be computed from the demands _l_ _kd_ and _w_ _kd_. We search
for the first yard \( i \), where the flow of the relation is split up and branch on the arc \((i, j)\) with the highest amount of flow of relation \( k_{\text{max}} \).

5 Cuts

Using the branching scheme mentioned above, it is possible to incorporate problem tailored cuts. With respect to constraints (4) and (5) it is obvious that trains are needed on an arc which is used by at least one relation. If \( x_{ij}^k \) is fixed to one, it is necessary to establish at least enough trains on \((i, j)\) to transport the wagons of \( k \). Denoting the set of relations which are forced to use \((i, j)\) by branching by \( K_{\text{fix}} \), we obtain the following cuts:

\[
n_{ij} \geq \max_{d \in D} \left( \frac{\sum_{k} l_{d}^{k} K_{\text{fix}}}{700}, \frac{\sum_{k} w_{d}^{k} K_{\text{fix}}}{1600} \right)_{ij}
\] (10)

Cuts (10) include the following well-known cuts (11) from multicommodity network design as a special case.

\[
n_{ij} \geq x_{ij}^k
\] (11)

6 Numerical Results

Combining BaP with the cuts (10), we obtained a Branch-and-Price-and-Cut (BaPaC) algorithm. BaPaC was implemented using CPLEX LP solver for the RMPs as described in [12]. We compare our BaPaC approach to CPLEX on the arc-based model \((OPT_{\text{arc}})\). The results can be found in Table 1. The instances are derived from scenarios provided by Deutsche Bahn. They vary in terms of network size and density of demands, i.e., the ratio of relations with demand and the number of OD-pairs in the network. That value is an indicator for the possibility of flow consolidation, because a higher density permits more synergetic effects between relations, see Graphic 2. The column ‘Root’ contains the gap resulting from BaPaC using only the paths generated in the root node of the tree. The columns ‘BaPaC’ and ‘CPLEX’ give the relative gaps of the best solutions obtained by our BaPaC and CPLEX 12.4, respectively. They are compared to the best lower bound (LB) provided by CPLEX. Due to its cutting planes, CPLEX mostly obtains tighter bounds than other approaches we tested. All computations were executed on a mod-
ern workstation computer.

Table 1: Computational results

<table>
<thead>
<tr>
<th>#nodes/ #relations of demands</th>
<th>density</th>
<th>time</th>
<th>Root</th>
<th>BaPaC</th>
<th>CPLEX</th>
</tr>
</thead>
<tbody>
<tr>
<td>30/65</td>
<td>7,4 %</td>
<td>7200s</td>
<td>2,00%</td>
<td>1,55%</td>
<td>1,76%</td>
</tr>
<tr>
<td>50/224</td>
<td>9,1 %</td>
<td>21600s</td>
<td>23,53</td>
<td>23,53%</td>
<td>no feas</td>
</tr>
<tr>
<td>21/254</td>
<td>60,5 %</td>
<td>21600s</td>
<td>21,97%</td>
<td>14,08%</td>
<td>14,21%</td>
</tr>
<tr>
<td>21/254</td>
<td>60,5 %</td>
<td>21600s</td>
<td>53,64%</td>
<td>25,38%</td>
<td>22,35%</td>
</tr>
</tbody>
</table>

Graphic 2: visualization of first and last instances’ solutions

7 Conclusions

We presented a model for a network design problem arising from railway freight traffic. Incorporating a column generation scheme into a branch-and-bound framework, we obtained a branch-and-price method. We discussed several ways of branching and traversing the tree. Moreover, we were able to derive cuts, which fit into the branching scheme. The solutions computed by our implementation could keep up with the solutions provided by CPLEX. Nevertheless, we are neither able to solve medium-sized instances to optimality nor to find feasible solutions for larger instances. Exact approaches are not appropriate for solving real-world instances. Hence, our future efforts will be concentrated on finding heuristic methods.

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References


