A planning and routing model for patient transportation in health care

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Abstract

In this paper, a problem concerning both the planning of health care services and the routing of vehicles, for patients transportation is addressed. An integrated approach, based on the column generation technique, is proposed to solve the planning and routing problem. Preliminary results on real data show the effectiveness of the proposed approach.

Keywords: Planning and routing, health care services, heuristic.

1 Introduction and problem description

In this paper, a problem concerning the planning of health care services and the transportation of patients, in not urgent conditions, is addressed. The addressed problem arises from a real world context, namely the health care system of Tuscany, an Italian region [1]. The transportation services are performed by non-profit organizations, by means of heterogeneous vehicles (e.g.,}

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ambulance, bus, car, etc.), located in geographically distributed depots. In this context, the transportation services include the transportation of patients for discharges, transfers and pickup from their home addresses to health care facilities for health care services, such as medical examinations, consultations, therapeutic treatments. The problem has a tactical and an operational dimension [1]. The operational dimension is to determine, on the basis of a given set of transportation requests and a timetable of the visits, a transportation plan for a set of patients from a set of origins to a set of destinations (and vice versa). The aim is of minimizing the total transportation costs, satisfying a set of constraints on the quality and the timing of the service offered. The operational plan of transportation must take into account of agreements, constraints and roles established between healthcare management and non-profit organizations, which perform the transportation services and have a certain degree of autonomy in decision-making. The tactical dimension of the problem aims to integrate the transportation planning, i.e., the routing of the vehicles, with the definition of the visits timetable, i.e., the time in which the health care service of each patient will start. In this case, the problem consists in determining a timetable of the health care services of the patients, in such a way that the transportation costs are minimized, always respecting service quality constraints. Each transportation request corresponds to a patient that must be carried on by a vehicle, from a pickup location to a delivery location, satisfying capacity, feasibility and time windows constraints. The capacity constraints are related to the number of available seats in the vehicle and to the number of seats occupied by each patient (e.g., patients may need a stretcher or a wheelchair to be transported). The feasibility constraints set the vehicle typology that can be used to serve a given patient typology, depending on the specific set up of the vehicle (for example, a patient may require to be transported by an ambulance). Time windows constraints are related to the patients, requiring that each pickup and delivery location need to be reached within a time interval, and depend on the timetable of the visits. In order to guarantee quality of service requirements, constraints on the lengths of the routes and on the travel and waiting times of each patient must be also considered.

The operational dimension of the problem we address in the paper is strictly related to a problem known in the literature as Dial-A-Ride Problem (DARP), [3,6,9,11], a generalization of the Pickup and Delivery Problem with Time Windows (PDPTW)[2,4,10]. In fact, the operational dimension of the problem can be defined as a Heterogeneous Multi-Depot Dial-A-Ride Problem. Both the DARP and PDPTW are $NP$-hard, therefore, the de-
development of solution methods for these problems have mainly focused on heuristics [5]. In some cases, the column generation technique has been successfully applied to develop exact and heuristic methods for these types of problems [2,4,7,8,10,12]. The above approaches from the literature are focused in solving the operational problem (i.e., considering only the routing aspects), while in this paper, we present an integrated approach for solving the tactical and operational dimension of the problem. More precisely, we propose an efficient heuristic algorithm based on a set covering formulation of the problem for finding both the visits timetable and the routing of the vehicles. Column generation is employed both to generate a set of possible routes and to plan the timetable of the health care services. Simulation results based on real world data arising from the healthcare Italian system show the effectiveness of the proposed approach.

The paper is organized as follows. In Section 2, a mathematical formulation and a solution approach for the tactical dimension of the problem are presented. Preliminary results are reported in Section 3.

2 Mathematical model and formulation

The addressed problem is characterized by two main sets of elements: the Vehicles Types and the Transportation Requests. In the following, the main characteristics of these two elements are listed.

Vehicle Type characteristics: depot location, capacity (i.e., the number of seats in the vehicle), type (ambulance, equipped vehicle, car or bus), number of available vehicles.

Transportation Request characteristics: pickup and delivery location, number of seats occupied on a vehicle, sets of possible pickup time and delivery time windows (depending on the health care service type), health care service type (e.g., transfer, treatment, etc.).

2.1 Problem formulation and solution approach

The addressed problem can be modeled as a set covering problem, as described in the following. Let $S$ be the set of health care service types. The service type defines both the service typology (e.g., discharge, treatment etc.) and the health care structure where the service is performed. Let $n'_s$ be the maximum number of health care services of type $s$ that can be allocated to time slot $t$ (in the health care structure associated to service $s$). Let $Q$ be the set of transportation requests and let $Q_s$ be the set of transportation requests.
of service type $s$, with $Q_s \subseteq Q$. Let $T_q$ be the set of time slots in which the health care service associated to the transportation request $q$ can be performed. The transportation requests in $Q$ are partitioned into: 1) *outbound* requests, 2) *inbound* requests and 3) *one-way* requests. We denote by $Q_A$ the set of the outbound requests, and given $q \in Q_A$, by $q_R$ the inbound request corresponding to $q$.

Let $V$ be the set of vehicle types, where the vehicle type defines both the vehicle typology and the depot in which the vehicle is located. Let $n_p$ be the maximum number of vehicles of type $p$ and let $\Omega_p$ be the set of routes that can be feasibly assigned to a vehicle of type $p$. Moreover, let $c^{k,p}$ be the cost of the route $k$ when assigned to vehicle type $p$. Given a route $k$, let $a^{l}_{q_k}$ be a coefficient equal to 1 if: (i) the route $k$ serves the transportation request $q$, and (ii) the health care service (associated to the transportation request $q$) is assigned to time slot $t$. In other words if $a^{l}_{q_k} = 1$, the patient corresponding to request $q$ is served by route $k$ and is delivered at her destination to perform the health care service at slot $t$. Let $z^{k,p}$ be a binary variable equal to 1 if the route $k$ is assigned to a vehicle of type $p$, then the problem can be formulated as follows.

$$\min \sum_{p \in V} \sum_{k \in \Omega_p} c^{k,p} \cdot z^{k,p}$$  \hspace{1cm} (1)

$$\sum_{p \in V} \sum_{k \in \Omega_p} \sum_{t \in T_q} a^{l}_{q_k} \cdot z^{k,p} \geq 1 \hspace{0.5cm} \forall q \in Q$$  \hspace{1cm} (2)

$$\sum_{k \in \Omega_p} z^{k,p} \leq n_p \hspace{0.5cm} \forall p \in V_p$$  \hspace{1cm} (3)

$$\sum_{p \in V} \sum_{k \in \Omega_p} \sum_{q \in Q_s} a^{l}_{q_k} \cdot z^{k,p} \leq n^t_s \hspace{0.5cm} \forall s \in S, \forall t \in \{T_q : q \in Q_s\}$$  \hspace{1cm} (4)

$$-\sum_{u \geq t} \sum_{p \in V} \sum_{k \in \Omega_p} a^{u}_{q_{R,k}} \cdot z^{k,p} \leq 0 \hspace{0.5cm} \forall q \in Q_A, \forall t \in \{T_q : q \in Q_s\}$$  \hspace{1cm} (5)

$$z^{k,p} \in \{0, 1\} \hspace{0.5cm} \forall p \in V, \forall k \in \Omega_p$$  \hspace{1cm} (6)

The objective function (1) accounts for the minimization of the overall selected routes. Constraints (2) impose that a request must be served by at most a route. Constraints (3) set the maximum number of routes that can be assigned to a vehicle type. Constraints (4) state that the number of health care services of type $s$ that can be assigned to time slot $t$ cannot be greater than $n^t_s$. Constraints (5) state that if an outbound request $q \in Q_A$ is assigned to time slot $t$ then its related return request $q_R$ must be assigned to a time slot $u \geq t$. The solution approach that we propose for problem (1)–(6) is a two-phase heuristic procedure, based on the column generation technique.
In the first phase of the heuristic, the linear relaxation of (1)–(6) is solved by a column generation approach, where the pricing problems are solved by heuristic algorithms. In the second phase, a branch-cut algorithm is employed for solving the problem obtained in the first phase. Cplex is used for solving the problem in the second phase. Observe that, since the solution (possibly, fractional) obtained in the first phase could not coincide with the optimal solution of the linear relaxation, the overall solution approach could not be able to find the optimal solution of the addressed problem.

In the first phase, a restricted master problem (RMP) is first determined, by including a set of routes, for which a feasible solution exists for the problem. Then, the new columns to add to the RMP are detected by heuristically solving a set of pricing problems, one for each vehicle type. The pricing problems consist in finding variables with negative reduced cost corresponding to the violated dual constraints, reported in (7), or prove they don’t exist.

\[
\sum_{q \in Q} \sum_{t \in T_q} a_{qk}^t \mu_q - \delta_p - \sum_{s \in S} \sum_{q \in Q_s} \sum_{t \in T_q} a_{qk}^t \gamma_{st} + \sum_{q \in Q_A} \sum_{t \in T_q} (-a_{qk}^I + \sum_{u \geq t} a_{qk}^u) \tau_{qt} \leq c_{k,p} (7)
\]

In (7), \(\mu_q\), \(\delta_p\), \(\gamma_{st}\) and \(\tau_{qt}\) are the dual variables related to constraints (2), (3), (4) and (5), respectively. The resulting pricing problems, one for each type of vehicle \(p \in V\), can be modeled as Resourced Constrained Elementary Shortest Path Problems on a graph \(G\), where the nodes of \(G\) correspond to patients locations, assistance structures locations and vehicle depots.

Given a vehicle of type \(p\), let \(y_q^t\) be a binary variable equal to 1 if: (i) the transportation request \(q\) is visited by the path served by a vehicle of type \(p\), and (ii) the health care service associated to request \(q\) is allocated at time slot \(t\); and 0 otherwise. Let \(c^p(y)\) be cost required by a vehicle of type \(p\) in order to satisfy the transportation requests \(q\) for which \(t \in T_q\) exists such that \(y_q^t = 1\). Let \(\mu^*_q\), \(\gamma^*_st\), \(\delta^*_p\) and \(\tau^*_qt\) be the optimal dual variables associated to RMP. Then the pricing problem related to a vehicle of type \(p\) can be formulated as the problem of finding a set of requests, defined by \(y\), that can be served by the vehicle and that minimize the following function:

\[
c^p(y) - \sum_{q \in Q} \sum_{t \in T_q} \mu^*_q y_q^t + \sum_{q \in Q_A} \sum_{t \in T_q} (\tau^*_q y_q^t - \sum_{u \geq t} \tau^*_q y_{qR}^u) + \delta^*_p + \sum_{s \in S} \sum_{q \in Q_s} \sum_{t \in T_q} \gamma^*_st y_q^t (8)
\]

In order to obtain a fast column generation procedure, we adopted a heuristic algorithm for solving the pricing problem that basically consists in a local search procedure based on best-insertion rules. The scheme of the heuristic for a vehicle of type \(p\) is roughly described in Algorithm 1. In the algorithm, let Route be a route iteratively constructed, starting at depot \(D_p\) and ending
at depot $D_p$ (where vehicles of type $p \in V_p$ are located), let $|\text{Route}|$ be its cardinality, and let $\alpha$ be an upper bound on the length of Route.

**Algorithm 1.** Heuristic algorithm for the Pricing Problem.

**Algorithm 1**

*Input:* $p \in V$;

*Output:* a set $\Phi$ of routes with negative (reduced) costs;

*Initialize* $\text{Route} = \{D_p, D_p\}$, and $\Phi = \emptyset$;

*while* $(|\text{Route}| < \alpha)$

  *If any, select the request $q \in \{Q \setminus \text{Route}\}$, such that the route $\{\text{Route} \cup q\}$ is feasible and $q$ can be inserted in $\text{Route}$ producing a maximum decreasing of the objective function (8);*

  *if* $(q$ exists) and (the cost of $\{\text{Route} \cup q\}$ is negative)

    *Set* $\text{Route} = \{\text{Route} \cup q\}$, save $\text{Route}$ in $\Phi$;

  *else* return $\Phi$, STOP;

*end while*

3 Computational results

In this section, the algorithm proposed in Section 2, in the following denoted as ColGen, is tested on 80 real-world instances arising from the Health Care System of Tuscany, Ausl 11 Empoli. The performance of ColGen are compared with both the real cost derived from the historical data and the solution obtained by a simple two-phase heuristic, working as follows. In the first phase of the heuristic, for each vehicle of type $p$, a set of routes that can be feasibly assigned to a vehicle of type $p$ are generated by a greedy procedure, based on best insertion rules. In the second phase, Cplex is used to select the best routes among those generated in the first phase. Two sets of 40 instances each, denoted as Set 1 and Set 2, have been considered that differ for the number of transportation requests that must be served. Set 1 contains instances with 40 requests and 40 vehicle types, and Set 2 contains instances with 50 requests and 40 vehicle types. Each set of instances is composed by 4 subsets (each containing 10 instances) denoted as $P(0), P(5), P(10), P(15)$. In particular, $P(0)$ contains instances in which all the visits timetables are fixed and known (on these instances the problem consists in solving only the operational dimension), while $P(i)$, with $i = 5, 10, 15$, contains instances in which the visit timetable must be determined for $i$ transportation requests (the other transportation requests in the instance have a known and fixed visit timetable).
The Tables 1 and 2 report the results for the instances in Set 1 and Set 2. Each row in the table reports the average values on the 10 instances of the related subset. In Tables 1 and 2, “Cost” denotes the total transportation cost of the solution found by ColGen, “Imp. ColGen” is the improvement (in %) of the solution found by ColGen with respect to the real cost derived from the historical data, “Imp. Heu” is the improvement (in %) of the solution found by the simple two-phase heuristic with respect to the real cost, “Col” denotes the total number of columns (routes) produced at the root node by ColGen, and t is the computation time in seconds required by the algorithm. In the tests, the upper bound α on the maximum length of a route is set to 12 in both the algorithms.

<table>
<thead>
<tr>
<th></th>
<th>Cost</th>
<th>Imp. ColGen</th>
<th>Imp. Heu</th>
<th>Col</th>
<th>t (sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>P(0)</td>
<td>687.73</td>
<td>56.22%</td>
<td>31.72%</td>
<td>449.40</td>
<td>1834.34</td>
</tr>
<tr>
<td>P(5)</td>
<td>680.44</td>
<td>57.35%</td>
<td>13.60%</td>
<td>500.00</td>
<td>1654.24</td>
</tr>
<tr>
<td>P(10)</td>
<td>670.55</td>
<td>57.46%</td>
<td>16.20%</td>
<td>590.00</td>
<td>2107.83</td>
</tr>
<tr>
<td>P(15)</td>
<td>668.18</td>
<td>57.65%</td>
<td>1.99%</td>
<td>1130.00</td>
<td>9893.57</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Cost</th>
<th>Imp. ColGen</th>
<th>Imp. Heu</th>
<th>Col</th>
<th>t (sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>P(0)</td>
<td>823.50</td>
<td>56.42%</td>
<td>34.49%</td>
<td>370.00</td>
<td>676.74</td>
</tr>
<tr>
<td>P(5)</td>
<td>812.53</td>
<td>57.05%</td>
<td>27.66%</td>
<td>680.00</td>
<td>3535.32</td>
</tr>
<tr>
<td>P(10)</td>
<td>805.69</td>
<td>57.40%</td>
<td>45.06%</td>
<td>833.33</td>
<td>6057.72</td>
</tr>
<tr>
<td>P(15)</td>
<td>794.08</td>
<td>62.28%</td>
<td>53.38%</td>
<td>1130.00</td>
<td>5683.44</td>
</tr>
</tbody>
</table>

In general, the results show that the solutions provided by our algorithm allow to reduce the total transportation cost of about 58% on average with respect to the real cost. Another aspect concerns the cost improvement obtained when the number of transportation requests with a not fixed visit timetable increases (in Set 2 an improvement of about 5%).
References


