Solving the Two-Stage Robust FTTH network design Problem under Demand Uncertainty

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Abstract

For the past few years, the increase in high bandwidth requiring services forced telecommunication operators like France Telecom - Orange to engage the deployment of optical networks, the Fiber To The Home Gigabit Passive Optical Network (FTTH GPON) technology, leading to new design problems. Such problems have already been studied. However, to the best of our knowledge, without taking into account the future demand uncertainty. In this paper, we propose a model for a two-stage robust optimization FTTH network design problem tackling the demand uncertainty. We propose an exact algorithm, based on column and constraint generation algorithms, and we show some preliminary results.

Keywords: Robust Optimization, Network Design, Mathematical Programming.
1 Introduction

Optimizing the fixed optical access network deployment is a priority for telecommunication operators. Let us present first the global scheme for the FTTH GPON technology which is mainly composed of optical fibers and splitters. A fiber entering a splitter is split into a given number of fibers of a higher level; this number is the splitter capacity. The level 1 fibers are originated from the source node, called Optical Line Terminal (OLT) and, for quality of service requirements, only the last level of fibers can supply the client demands. The problem is to locate the splitters and to determine the route of the fibers in order to serve given demands, while minimizing the overall cost of both installed splitters and fibers. One can find complete descriptions and models for this problem in [11] or in [13]. In [12], the impact of the solution in terms of future maintenance cost is also examined.

However, the demand is uncertain and considering the low precision one can have on it, we decided to tackle the problem with robust optimization, and more precisely, with two-stage robust optimization [5,6,14]: decisions are taken in two stages. First stage variables correspond to decisions taken before discovering the actual values of uncertain data, second stage variables are determined once uncertainty has been revealed, this is the recourse problem. Moreover, robust optimization allows one to handle problems without making a lot of assumptions on random data distribution, contrary to stochastic optimization. It tries to provide acceptable solutions, no matter what happens in the future, i.e. whatever the scenario that occurs.

This approach is very conservative because the cost one pays to protect himself against very unlikely scenarios may be seen as too prohibitive. But in fact, the marketing service at France-Telecom Orange can ”expect a given percentage of the total household number in an area to purchase our offer for optical fiber”. That allows us, following the ideas of [8,17], to bound the total scale deviation of the uncertain data.

Demand uncertainty leads to problem formulations with right-hand-side uncertainty, a specific and hard version of robust optimization problems: see [2,9,16] for general models and [1,3,4,10,15] for network design problems. All these papers consider continuous recourse variables, but there is very few works available on problems like ours, with integral recourse variables [7,18].

In this paper, we solve a two-stage robust optical network design problem with integral recourse variables. We first introduce a model for this problem, then we propose an algorithm inspired from [18] to solve it to optimality and we finally examine some experimental results.
2 Model for the two-stage robust problem

2.1 The deterministic problem

We briefly describe the deterministic version of the problem. The given infrastructure is represented by an undirected graph \( G = (V,E) \), \( a_{max} \) denotes the demand on each node \( i \in V \) and \( a_{max} \) the corresponding vector (the vectors are written in bold). There are \( K \) levels of splitters (\( K+1 \) levels of fibers), \( m^k \) denotes the capacity of a level \( k \) splitter and \( C^k \) its cost. The cost of a level \( k \) fiber along an edge \((i,j) \in E\) is denoted by \( d^k_{ij} \). Note that in reality, splitter cost is much more important than fiber cost. The set of solutions that satisfy both a given demand vector \( a \) and the architecture requirements is denoted by \( P_G(a) \). The variables are the number of level \( k \) splitters to install on node \( i \in V \), denoted by \( z^k_i \), and the number of level \( k \) fibers routed along an edge \((i,j) \in E\) from \( i \) to \( j \), denoted by \( f^k_{ij} \). The fibers have to be be installed in existing ducts that are considered large enough whatever the routing. Then, the deterministic problem \( PON^{det} \), without capacity constraints, may be written as such:

\[
PON^{det} = \min_{(f,z) \in P_G(a)} \sum_{i \in V} \sum_{k=1}^{K} C^k z^k_i + \sum_{(i,j) \in E} \sum_{k=1}^{K+1} d^k_{ij} \left( f^k_{ij} + f^k_{ji} \right) \quad \text{with}
\]

\[
\begin{align*}
\sum_{j \in \{(i,j) \in E\}} (f^1_{ji} - f^1_{ij}) &\geq z^1_i, \forall i \in V \setminus \{0\} \quad (0 = OLT) \\
\sum_{j \in \{(i,j) \in E\}} (f^k_{ji} - f^k_{ij}) + m^{k-1} z^{k-1}_i &\geq z^k_i, \forall i \in V, \forall k = 1..K \\
\sum_{j \in \{(i,j) \in E\}} (f^{K+1}_{ji} - f^{K+1}_{ij}) + m^K z^K_i &\geq a_i, \forall i \in V \\
z^k_i &\in \mathbb{N}, \forall i \in V, \forall k = 1..K \\
f^k_{ij} &\in \mathbb{R}^+, \forall (i,j) \in E, \forall k = 1..K
\end{align*}
\]

These constraints enable the fiber flow to be coherent with installed splitters in the network and with the client demands (see [11] for a complete description).

2.2 The two-stage robust problem

In reality, the future demand is greatly uncertain and it is highly expectable that not every household will purchase the fiber offer of a single telecommunication operator. Therefore, considering the very high amount of necessary investments, it was decided that the initial optical network would be designed to serve only a part of households. But, once the network is deployed, it is not acceptable for operational units to add or move fibers in order to supply
clients that were not connected to the network. However, adding or removing splitters is much more acceptable and the splitter cost is the major part of the investment [11]. That is why it was decided to route the fibers in order to serve each household but to install the splitters and light the fibers only once the actual demand is known.

The question is then: “how should one design the fiber network so that its worst future splitter cost may be the lowest possible?”

Knowing the number of potential subscribers at each node \( i \), denoted by \( a_{i}^{\text{max}} \), it is rather simple for marketing services to estimate what could be the maximum number of clients in the whole studied area. Let us denote it by \( \bar{A} \) (\( 0 \leq \bar{A} \leq \sum_{i \in V} a_{i}^{\text{max}} \)). Then, the uncertainty set containing the random demand vector \( a \), denoted by \( \Lambda \), is defined as such:

\[
\Lambda = \left\{ a \in \mathbb{N}^{\left| V \right|} \left| \begin{array}{l}
    a_i \leq a_{i}^{\text{max}}, \forall i \in V \\
    \sum_{i \in V} a_i \leq \bar{A}
  \end{array} \right. \right\} = \{ a^s, s = 1, \ldots, S \}
\]

where \( S \) is the total number of scenarios and \( a^s \) denotes a scenario. As in section 2.1, let us denote by \( f_{ij}^k \) the number of level \( k \) fibers routed along the edge \( (i, j) \) before the demand is revealed (i.e. so that every household is connected to the OLT) and by \( z_{i}^{k} \) the maximum number of level \( k \) splitters that node \( i \) can receive (i.e. the number of splitters one would install if all \( f_{ij}^k \) were used): \( f_{ij}^k \) and \( z_{i}^{k} \) are the first stage variables. The variables \( \zeta_{i}^{k} \) concern the number of level \( k \) splitters that will be installed on node \( i \), and \( \varphi_{ij}^{k} \) the number of level \( k \) fibers that will be lighted along the edge \( (i, j) \), once the demand is revealed. Then, the two-stage robust problem \( PON^{\text{rob}} \) may be written as such:

\[
PON^{\text{rob}} = \min_{(f,z) \in P_G(a_{\text{max}})} \left( \sum_{(i,j) \in E} \sum_{k=1}^{K+1} d_{ij} \left( f_{ij}^k + f_{ji}^k \right) + \max_{a \in \Lambda} \min_{(\varphi,\zeta) \in P_G(a)} \sum_{i \in V} \sum_{k=1}^{K} c_{k} \zeta_{i}^{k} \right)
\]

\[ \zeta \leq z, \varphi \leq f \]

### 3 Column-and-Constraint generation algorithm

We solve the two-stage robust problem \( PON^{\text{rob}} \) using the column-and-constraint generation algorithm of Zhao and Zeng [18] which is a method dedicated to robust problem with mixed integer recourse variables. We adapt this method
in the context of our problem. The problem $PON^{rob}$ has some characteristics. First it satisfies the full recourse property that is the recourse problem (i.e. the internal minimization problem) has a feasible solution for any value of the higher level variables $z$, $f$ and the demand vectors $a$. Next the continuous fiber variables $\varphi$ have a null cost in the objective function of the recourse problem. These characteristics are fully used to solve $PON^{rob}$. Now we shortly describe the main ideas of our method.

The uncertainty set contains a finite number $S$ of elements denoted by $\Lambda = \{a^s\}_{s=1}^S$. To each $a^s$ is associated a splitter recourse variable $\zeta^s$ and a fiber recourse variable $\varphi^s$. For shortening, we denote $C\zeta = \sum_{i\in V} \sum_{k=1}^K C^k \zeta^i_k$.

By enumerating all possible assignments of $a$, i.e. all scenarios of demands, the problem is written:

$$\min \eta, (f, z), (\zeta^s, \varphi^s) \sum_{(i,j) \in E} \sum_{k=1}^{K+1} d_{ij} \left( f_{ij}^k + f_{ji}^k \right) + \eta$$

with

$$\begin{align*}
\eta &\geq C\zeta^s, s = 1, ..., S \\
(\zeta^s, \varphi^s) &\in P_G(a^s), s = 1, ..., S \\
\zeta^s &\leq z, \varphi^s \leq f, s = 1, ..., S \\
(f, z) &\in P_G(a^{max})
\end{align*}$$

As $S$ may be huge, only a subset of constraints is used through a constraint generation algorithm. The sub-problem for finding a cut is:

$$Q(f, z) = \max \min_{a \in \Lambda} (\zeta, \varphi) \in P_G(a), \zeta \leq z, \varphi \leq f C\zeta$$

The solution of this problem gives a new scenario of demand that must be added as a constraint if $\eta < Q(f, z)$ otherwise the algorithm stops and $PON^{rob}$ is solved.

$Q(f, z)$ is written by enumerating all possible assignments of the integer splitter variables $\zeta$ in the internal minimization problem that will be generated through another column and constraint generation algorithm. Because of the article size limitations, we do not give details of the solving method for the subproblem $Q(f, z)$, since it is much more complex in its refinements yet very similar to the method used to solve the master problem. We advise the reader to refer to [18].
4 Experimental results

The method proposed to solve the robust problem involves a huge number of sub-problems to solve since there are two interlinked constraints generation algorithms. Then we can solve only small instances of the problem. Nevertheless, the interest of our tests is to emphasize the economic interest of a robust approach. Graphs were first randomly generated as tree-graphs to which we added a very few number of random edges between sons in order to obtain graphs topology close to our real-world ones. Tests were performed on more than 100 instances of various sizes (from 5 to 10 nodes) and a mean node degree of 3 (which is compliant with real world instances). Behaviours presented here were always qualitatively observed in our tests.

Fig. 1. Cost of the robust solution, compared to a 100% demand solution (the lowest curve is for the worst splitter cost).

Fig. 2. Comparison of the mean splitter cost over 2000 random scenarios between the trivial solution and the two-stage robust solution.

Figures 1 and 2 are extracted from results computed on a small instance of 5 nodes. Without any algorithmic improvement, we were not able to solve instances to optimality with 10 nodes or more. However, the behaviour shown in this paper has always been observed. Figure 1 assesses the cost improvement of the robust solution in function of the size of the uncertainty set (we consider various proportions of the total demand for the maximum demand possible in the area). The cost is expressed as a percentage of the One-Stage robust solution cost, which is computed by solving $\text{PON}^{\text{det}}$ for the demand vector $a_{\text{max}}$. The step-like aspect of the lowest curve is due to the fact that splitters are much more costly than fibers, and adding one splitter in the worst-case scenario implies a huge cost step. Of course, as the uncertainty set size increases, costs of both solutions tend to be equal.

Robust optimization deals with the worst case, but it is quite unlikely that every scenario will be a bad one. Figure 2 shows how effective is the two-stage
robust solution on any scenario compared to a trivial solution where no two-stage robust optimization has been made (here, we take the $PON^{det}$ problem solution with $a^{max}$). We randomly draw 2000 scenarios in the uncertainty set, and we compute the best splitter cost, given $f$ and $z$, for those scenarios. The figure shows that the mean cost of the robust solution is always better than the one of the trivial solution. This was not granted since we only optimize the worst-case splitter cost.

5 Conclusion

This paper introduced a model for the Passive Optical Network design with demand uncertainty, which was tackled with robust optimization techniques. The problem is solved to optimality through a complex column-and-constraints generation algorithm, whose complexity makes real-life instances too big to be solved. However, obtaining optimal robust solutions for this problem enabled us to show the interest of having a robust approach for solving this problem, especially compared to stochastic programming approaches since our tests showed that we tend to optimize the cost of all scenarios, not only the worst-case one. Moreover, as we do not need to consider probability laws for the demand uncertainty that we do not have in practice, robust optimization seems really suited for this particular problem. Future research steps are to draw conclusions from the analysis of robust optimal solutions in order to design heuristics, or local moves that may improve a solution’s robustness.

References


